Mechatronics Engineering Department
Faculty of Engineering
Ain Shams University

MCT344/MCT342/CSE373/CSE471: Robotics

Lecture 9: Dynamics I

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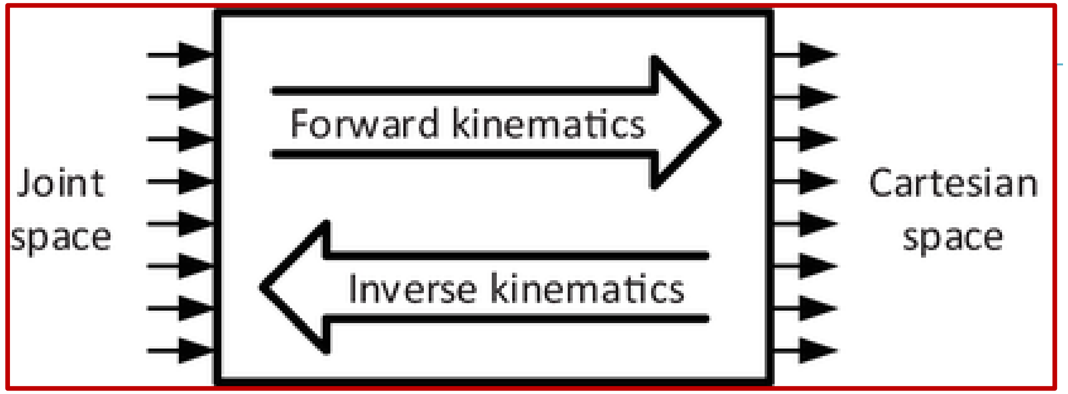
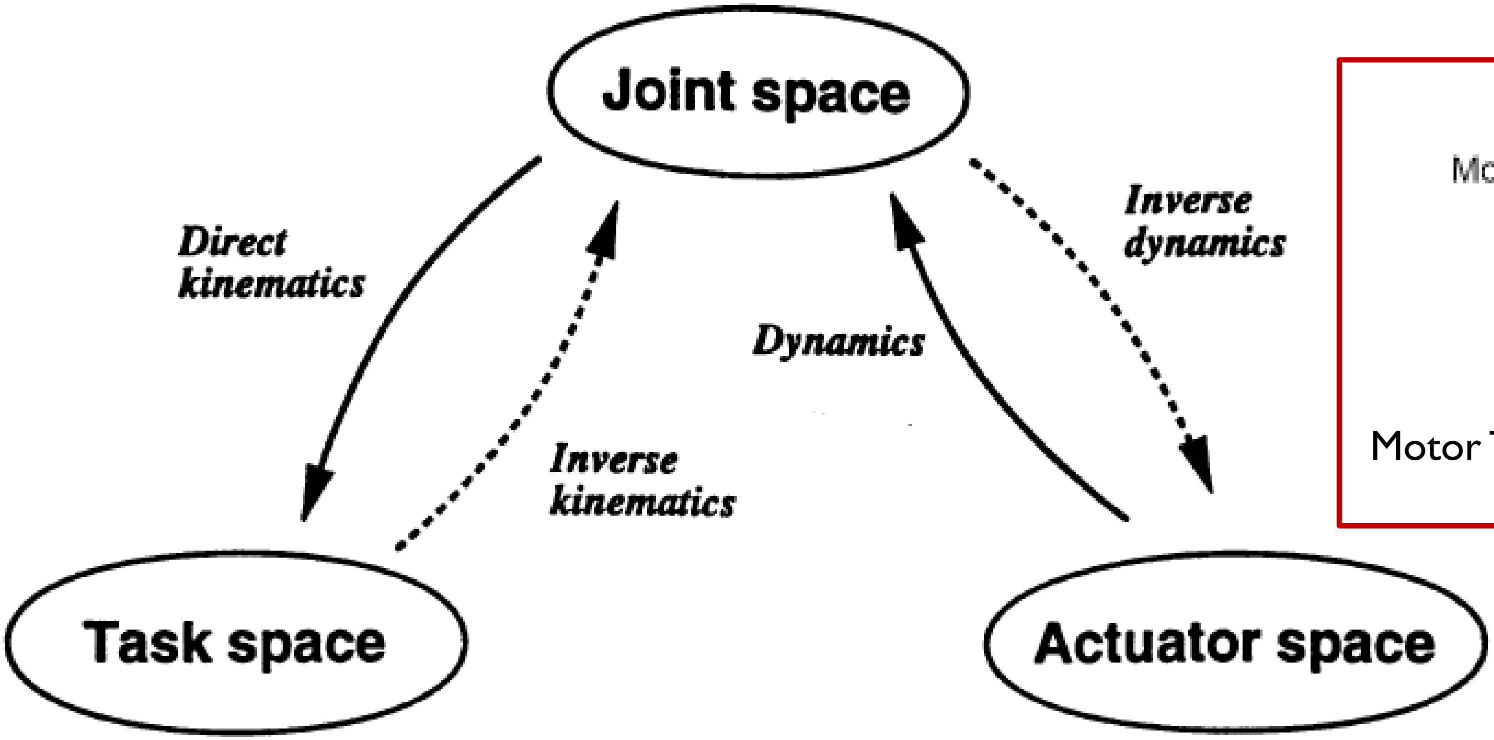
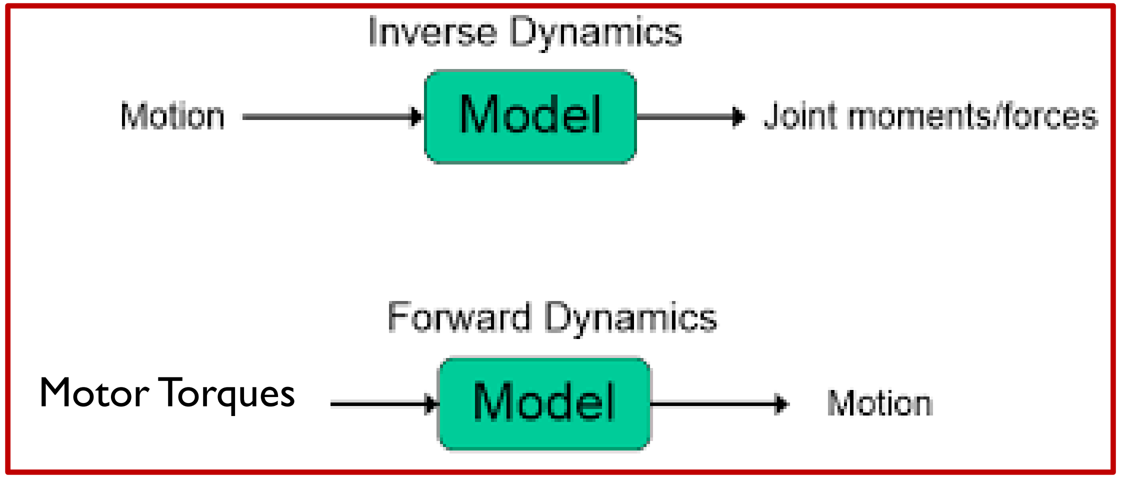
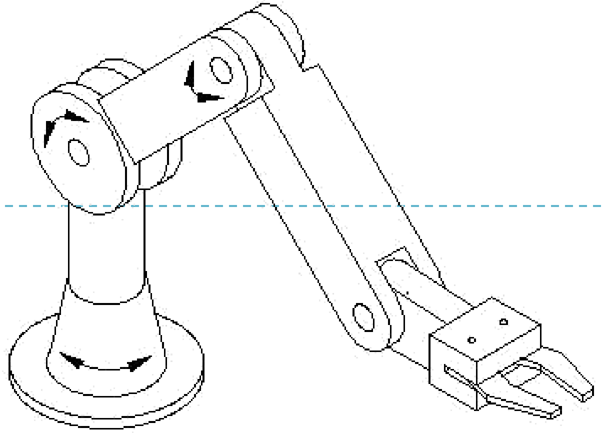
Spaces in which motion can be observed and mappings between them



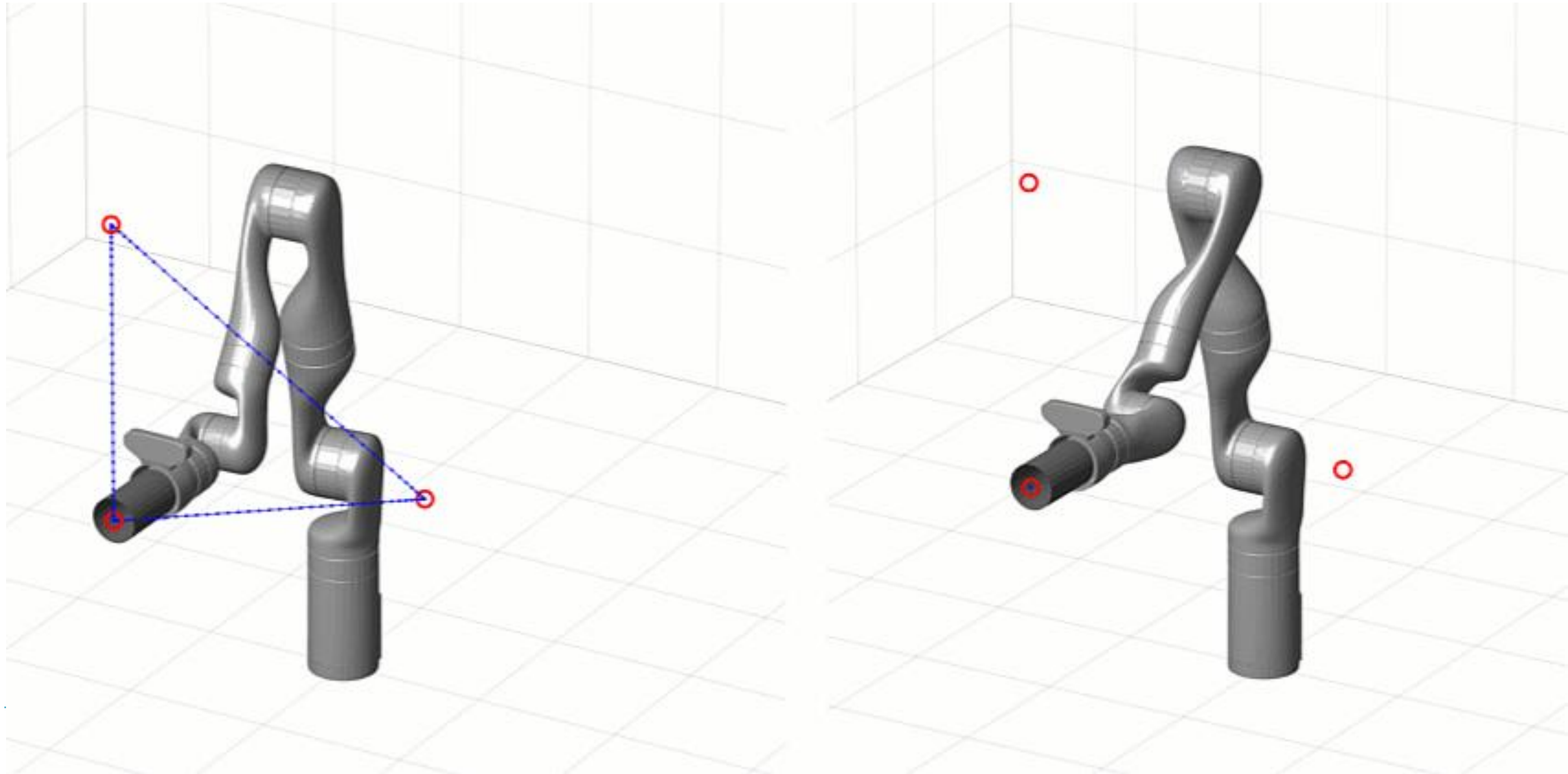




Originally a problem from robotics



- Given a robotic arm with multiple motor-actuated joints, determine the joint torques required to drive the arm from its initial state to a desired configuration or trajectory.



Why are we studying dynamics modelling?

Kinematic vs dynamic models:

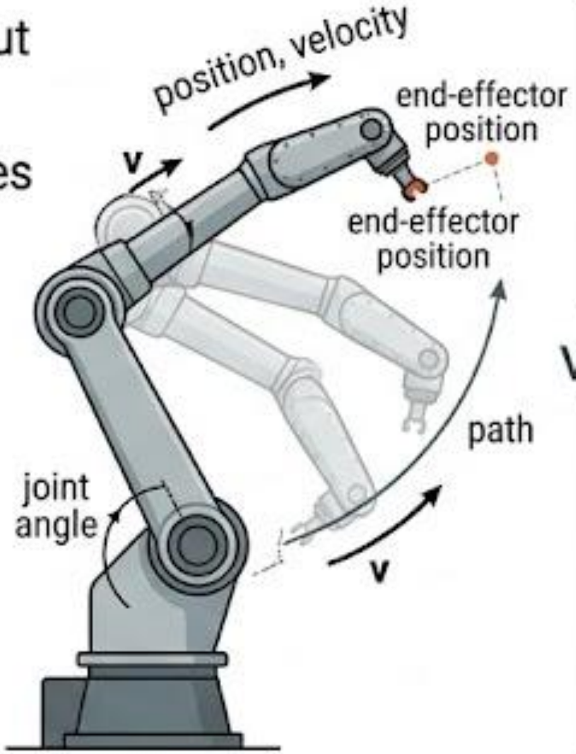
- What we're really doing is modeling the manipulator
- **Kinematic models**
  - The study of motion without regard to forces
  - Simple control schemes
  - Good approximation for manipulators at low velocities and accelerations when inertial coupling between links is small
  - Not so good at higher velocities or accelerations
- **Dynamic models**
  - The study of motion with regard to forces. The study of the relationship between forces/torques and motion. Composed of kinematics and kinetics
  - More complex controllers
  - More accurate

Kinematic vs Dynamic Models of a Manipulator

What we are really doing is modeling the manipulator

Kinematic Models

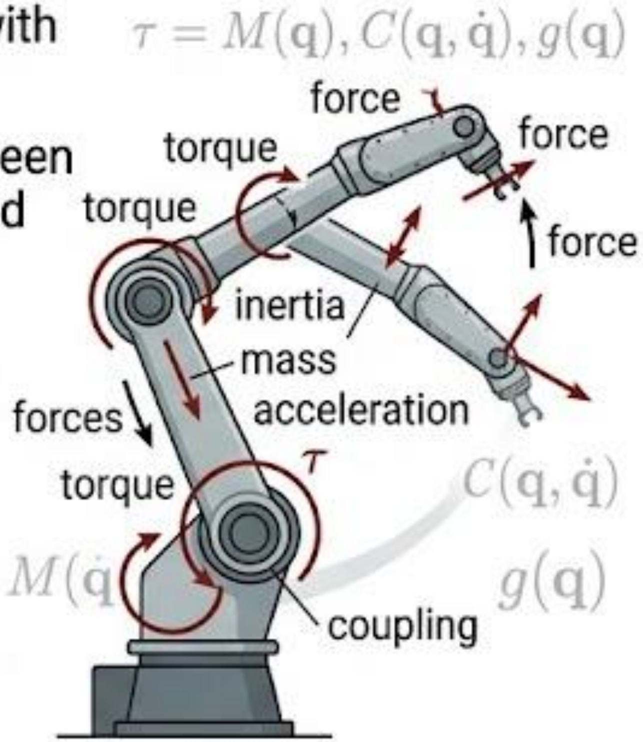
- Study of motion without regard to forces
- → Simple control schemes
- → Good approximation for manipulators at low velocities and accelerations
- → Valid when inertial coupling between links is small
- → Less accurate at higher velocities or accelerations



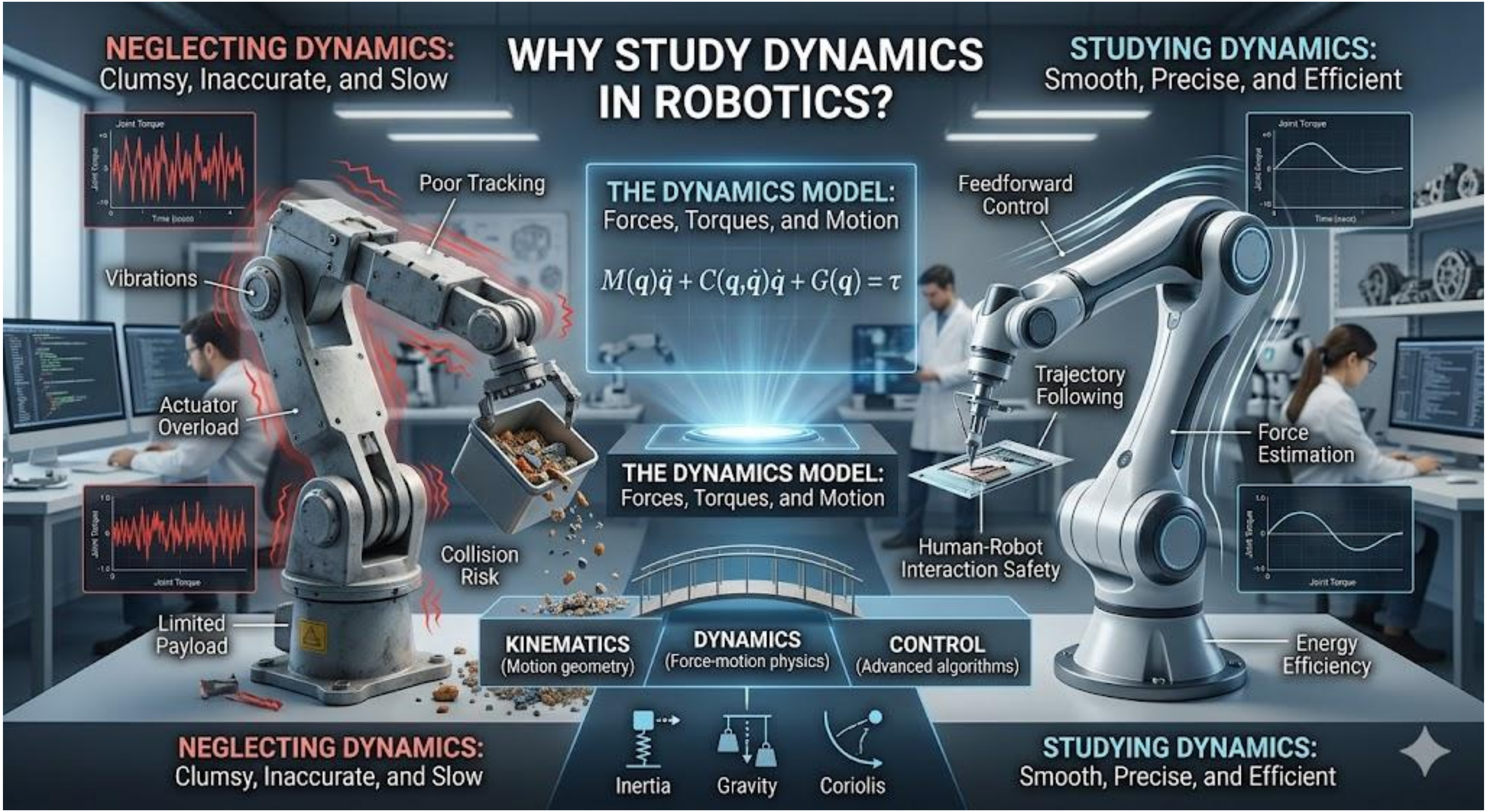
vs

Dynamic Models

- Study of motion with regard to forces
- → Relationship between forces/torques and motion
- → Composed of kinematics and kinetics
- → More complex controllers
- → More accurate

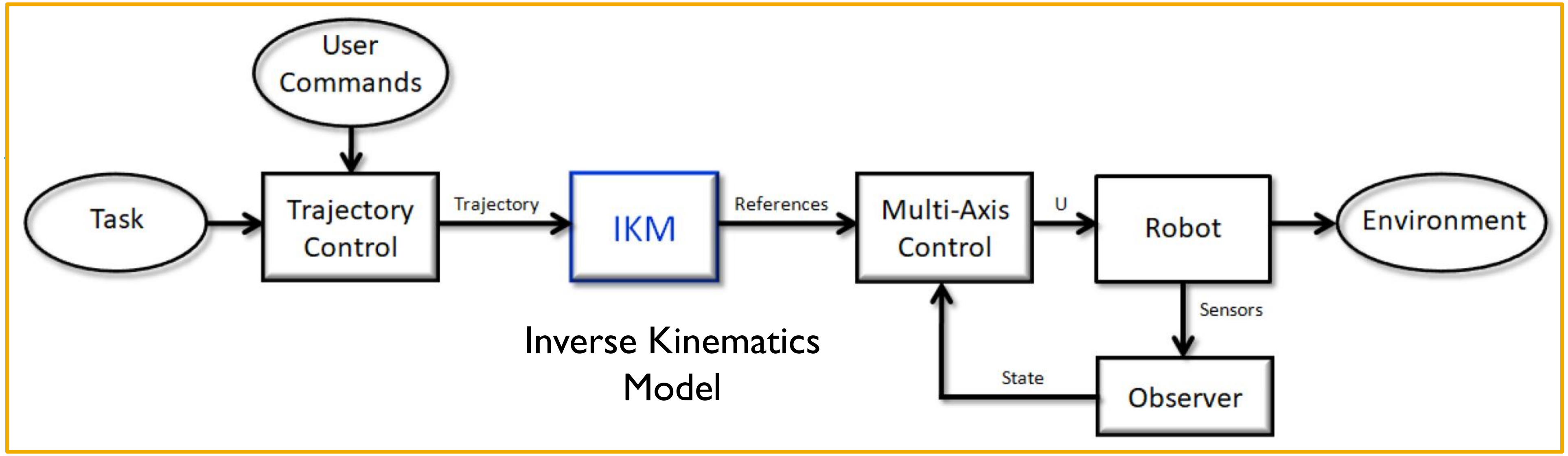


Kinematics = easier, simpler, faster
Dynamics = richer, more accurate, force-aware

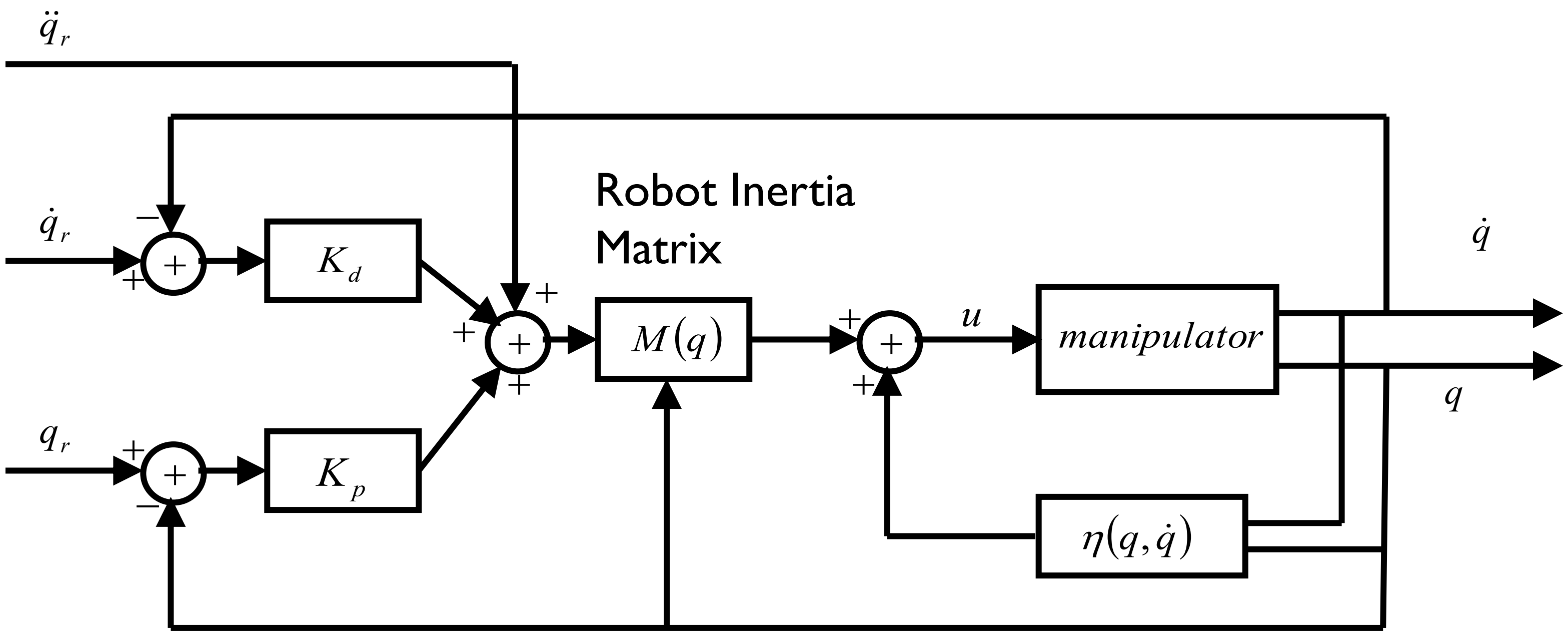


WHY STUDY DYNAMICS IN ROBOTICS?
NEGLECTING DYNAMICS:
Clumsy, Inaccurate, and Slow
STUDYING DYNAMICS:
Smooth, Precise, and Efficient
Joint Torque
Poor Tracking
Vibrations
Actuator Overload
Collision Risk
Limited Payload
THE DYNAMICS MODEL:
Forces, Torques, and Motion
$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$
THE DYNAMICS MODEL:
Forces, Torques, and Motion
Feedforward Control
Trajectory Following
Force Estimation
Human-Robot Interaction Safety
Energy Efficiency
KINEMATICS
(Motion geometry)
DYNAMICS
(Force-motion physics)
CONTROL
(Advanced algorithms)
Inertia
Gravity
Coriolis
NEGLECTING DYNAMICS:
Clumsy, Inaccurate, and Slow
STUDYING DYNAMICS:
Smooth, Precise, and Efficient

Simple control based on kinematics



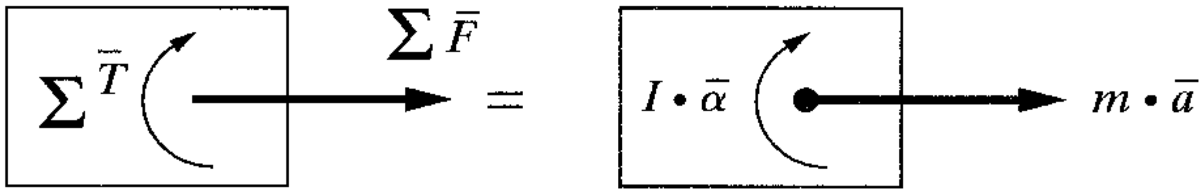
Control based on Dynamics



Dynamic model

- The dynamics, related with accelerations, loads, masses and inertias.

$$\sum \overline{F} = m \cdot \overline{a} \qquad \sum \overline{T} = I \cdot \overline{\alpha}$$



Force-mass-acceleration and torque-inertia-angular acceleration relationships for a rigid body.

In Actuators…….

- The actuator can be accelerate a robot's links for exerting enough forces and torques at a desired acceleration and velocity.
- By the dynamic relationships that govern the motions of the robot, considering the external loads, the designer can calculate the necessary forces and torques.

Dynamic model

- **Dynamic models**
  - Forward/Direct Dynamics (Simulation):

    Give the actuator forces and torques compute the resulting motion. Give τ calculate θ , $\dot{\theta}$, $\ddot{\theta}$
  - Inverse Dynamics (Control & Biomechanics Analysis):

    Give the **desired**, calculate the actuator forces and torques. Give θ , $\dot{\theta}$, $\ddot{\theta}$ calculate τ

Dynamic Modelling Methods

- **Inverse Dynamics/ Forward Dynamics**
  - Euler-Lagrange (energy based/principle of virtual work)
  - Newton-Euler Algorithm

Approaches to dynamic modelling

energy-based approach (Euler-Lagrange) ⇌ **Newton-Euler method (balance of forces/torques)**

- multi-body robot seen as a whole
- constraint (internal) reaction forces between the links are automatically eliminated: in fact, they do not perform work
- closed-form (symbolic) equations are directly obtained
- best suited for study of dynamic properties and **analysis** of control schemes

- dynamic equations written separately for each link/body
- inverse dynamics in real time
  - equations are evaluated in a numeric and recursive way
  - best for **synthesis** (=implementation) of model-based control schemes
- by elimination of reaction forces and back-substitution of expressions, we still get closed-form dynamic equations (identical to those of Euler-Lagrange!)

Euler–Lagrange vs Newton–Euler Dynamic Analysis

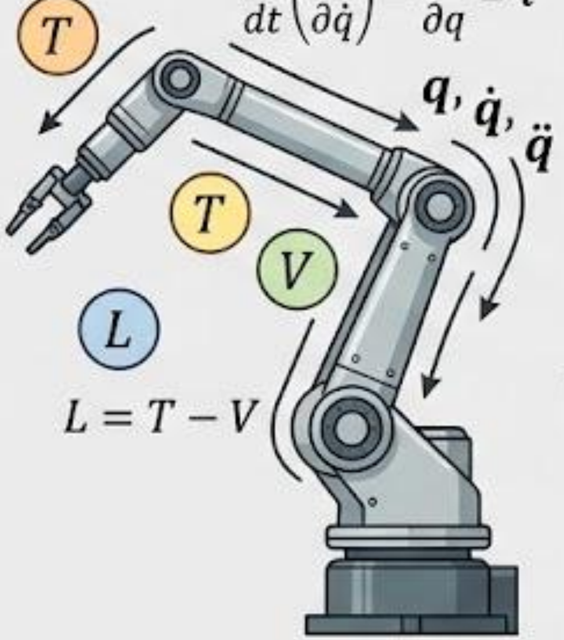
Two equivalent ways to model robot manipulator dynamics

Euler–Lagrange Method

Energy-based approach

- Models the multi-body robot as a whole
- Based on kinetic and potential energy
- Internal constraint reaction forces are automatically eliminated
- Produces closed-form symbolic equations
- Best suited for **dynamic analysis** and theoretical study
- Convenient for studying system properties, stability, and control analysis
- Can become computationally heavy for high-DOF robots

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau$$



$$L = T - V$$

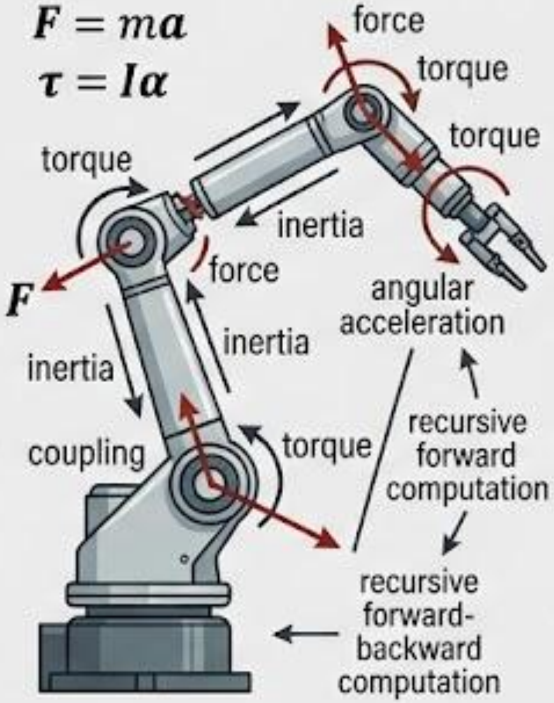
=

Different formulation, same robot dynamics

Newton–Euler Method

Force/Torque balance approach

$$\boldsymbol{F} = m\boldsymbol{a}$$
$$\boldsymbol{\tau} = \boldsymbol{I\alpha}$$



- Dynamic equations written separately for each link
- Based on balance of forces and torques
- Excellent for **inverse dynamics**
- Efficient recursive numeric computation
- Best suited for **real-time implementation** and model-based control
- Well suited for robots with many links
- Internal forces appear during formulation but are eliminated by substitution

Euler–Lagrange: best for symbolic modeling and analysis
Newton–Euler: best for efficient computation/ real-time control
Both can yield the same manipulator dynamic equations

Euler-Lagrange method (energy-based approach)

basic assumption: the N links in motion are considered as **rigid bodies**
(+ possibly, **concentrated elasticity** at the joints)

$q \in \mathbb{R}^N$ generalized coordinates (e.g., joint variables, but not only!)

Lagrangian $\boxed{L(q,\dot{q}) = T(q,\dot{q}) - U(q)}$

kinetic energy – potential energy

- least action principle of Hamilton
- virtual works principle

Euler-Lagrange equations

$$\boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = u_i} \qquad i = 1, \ldots, N$$

non-conservative (external or dissipative)
generalized forces performing work on q_i

Calculating Equations of Motion using the Lagrangian

$$L = T - U$$

Kinetic energy

Potential energy

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i$$

Generalized coordinate
(joint angle)

Generalized force
(torque)

Example

Derive the force-acceleration relationship for the one-degree of freedom system.

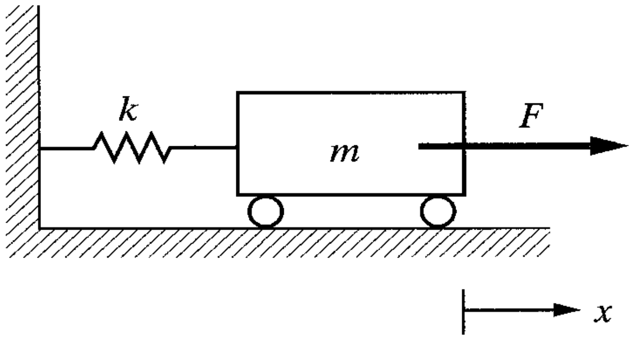


Fig. 4.2 Schematic of a simple cart-spring system.

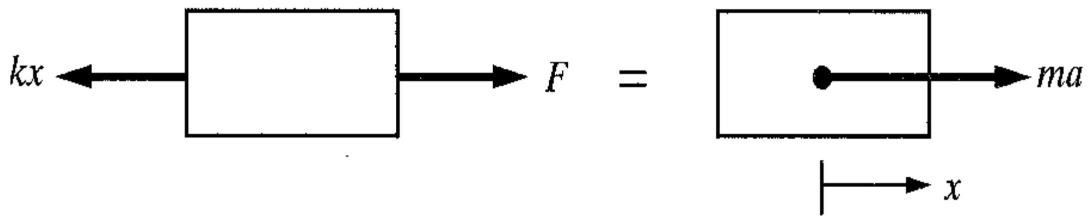


Fig. 4.3 Free-body diagram for the sprint-cart system.

Solution

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{x}^2, P = \frac{1}{2} k x^2$$

$$L = K - P = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$$

- Lagrangian mechanics

$$\frac{\partial L}{\partial \dot{x}_i} = m\dot{x}, \frac{d}{dt}(m\dot{x}) = m\ddot{x}, \frac{\partial L}{\partial x} = -kx$$

$$F = m\ddot{x} + kx$$

- Newtonian mechanics

$$\sum \overline{F} = m \cdot \overline{a}$$

$$F - kx = ma \rightarrow F = ma + kx$$

- The complexity of the terms increases as the number of degrees of freedom and variables.

Example: pendulum

Earlier, we derived the equation of motion of the inverted pendulum by summing the torques:

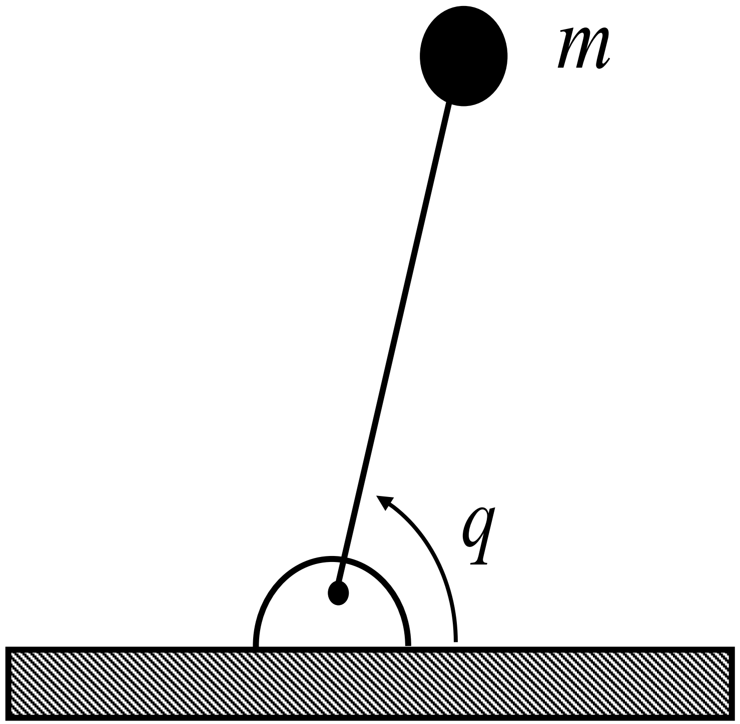
$$0 = -ml^2\ddot{q} - mgl\cos(q)$$

You can also do it using the Lagrangian:

$$L = T - U$$

$$T = \tfrac{1}{2} I\dot{q}^2 = \tfrac{1}{2} ml^2\dot{q}^2$$

$$U = mgl\sin(q)$$



Example: pendulum

$$L = T - U$$

$$T = \tfrac{1}{2} I\dot{q}^2 = \tfrac{1}{2} ml^2\dot{q}^2$$

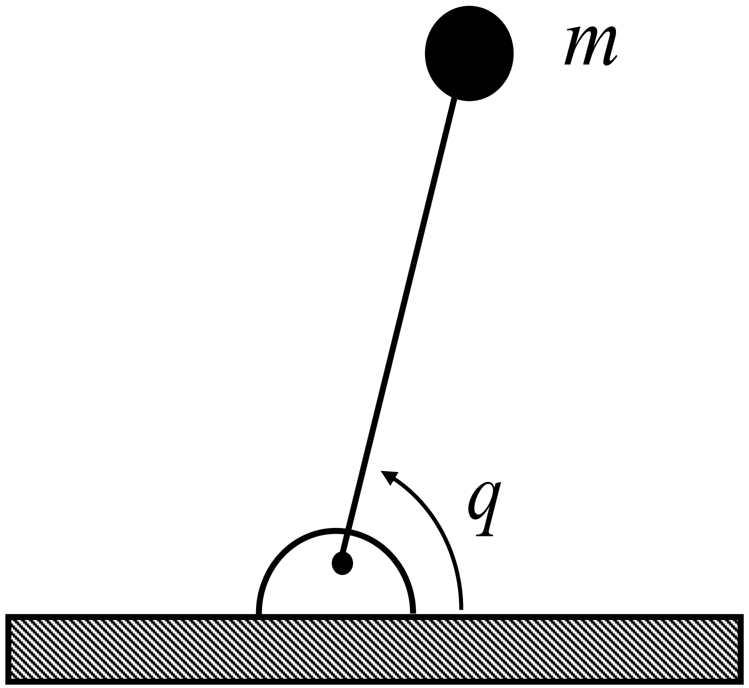
$$U = mgl\sin(q)$$

$$L = T - U = \tfrac{1}{2} ml^2\dot{q}^2 - mgl\sin(q)$$

$$\frac{\partial L}{\partial \dot{q}} = ml^2\dot{q} \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = ml^2\ddot{q} \qquad \frac{\partial L}{\partial q} = -mgl\cos(q)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau$$

$$ml^2\ddot{q} + mgl\cos(q) = 0$$



Example: pendulum

w/ torque and friction:

$$ml^2\ddot{q} + mgl\cos(q) = \tau - F\dot{q}$$

Torque applied at motor

Coefficient of friction

Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = u_i \qquad i = 1, \ldots, N$$

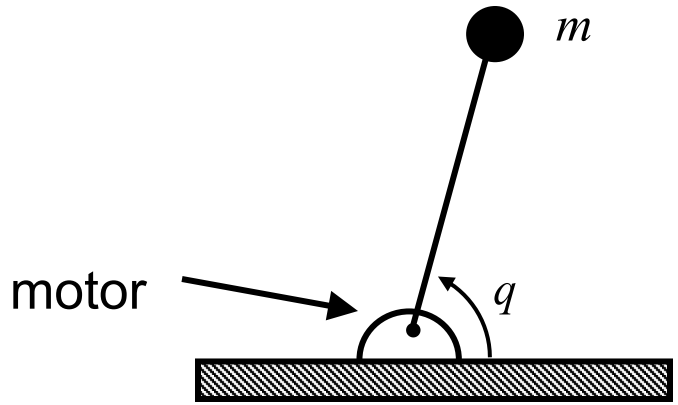
non-conservative (external or dissipative) generalized forces performing work on q_i

If you want to think about a motor actuating the joint:

$$T = \tfrac{1}{2} I_l \dot{q}^2 + \tfrac{1}{2} I_m (k_r \dot{q})^2 = \tfrac{1}{2} ml^2 \dot{q}^2 + \tfrac{1}{2} I_m k_r^2 \dot{q}^2$$

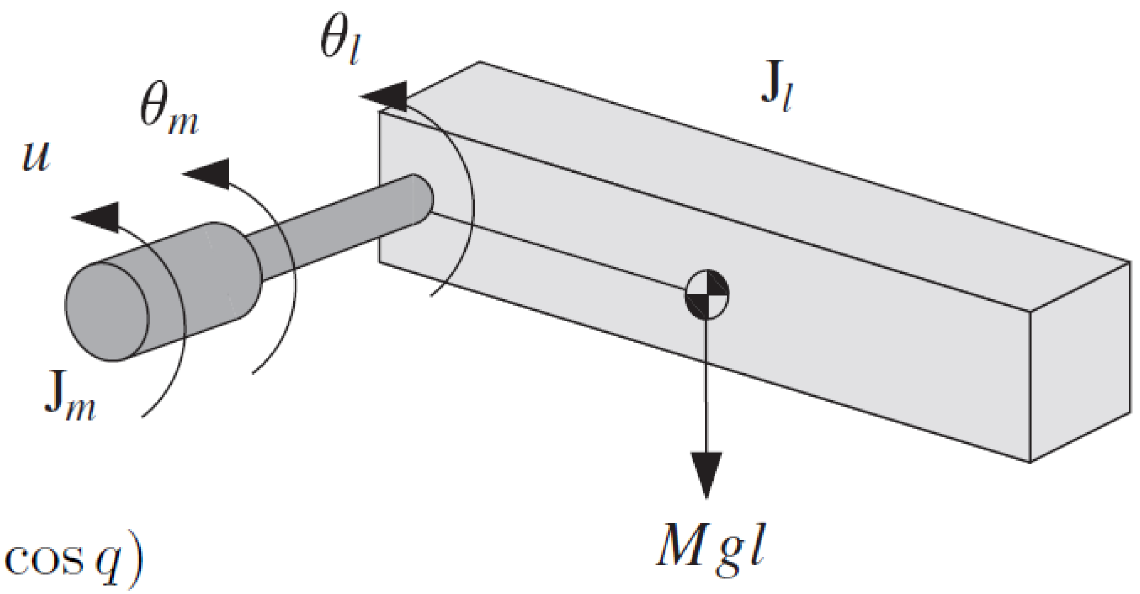
motor gear reduction

$$\left(ml^2 + I_m k_r^2\right)\ddot{q} + mgl\sin(q) = \tau - F\dot{q}$$



Example 6.1 (Single-Link Manipulator). *Consider the single-link robot arm shown in Figure 6.2, consisting of a rigid link coupled through a gear train to a DC motor. Let θ_ℓ and θ_m denote the angles of the link and motor shaft, respectively. Then, $\theta_m = r\theta_\ell$ where $r : 1$ is the gear ratio. The*

$$\begin{aligned} K &= \frac{1}{2}J_m\dot{\theta}_m^2 + \frac{1}{2}J_\ell\dot{\theta}_\ell^2 \\ &= \frac{1}{2}(r^2 J_m + J_\ell)\dot{q}^2 \end{aligned}$$



Defining $I = r^2 J_m + J_\ell$,

$$P = Mg\ell(1 - \cos q)$$

$$L = K - P \Longrightarrow \mathcal{L} = \frac{1}{2}I\dot{q}^2 - Mg\ell(1 - \cos q)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \Longrightarrow I\ddot{q} + Mg\ell\sin q = \tau_\ell$$

input motor torque $u = r\tau_m \Longrightarrow \tau_\ell = u - B\dot{q} \Longrightarrow I\ddot{q} + B\dot{q} + Mg\ell\sin q = u$

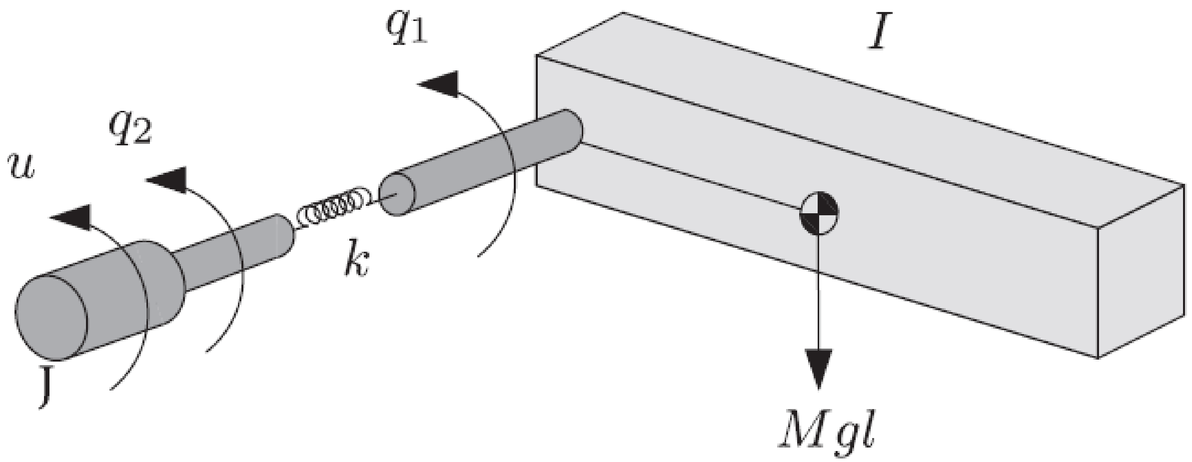
Example 6.2 (Single-Link Manipulator with Elastic Joint). *Next, consider a single-link manipulator including the transmission flexibility as shown in Figure 6.3.*

In this case the motor angle $q_1 = \theta_\ell$ and the link angle $q_2 = \theta_m$ are independent variables and so the system possesses two degrees of freedom. Thus, two generalized coordinates are required to specify the configuration of the system.

The kinetic energy of this system is

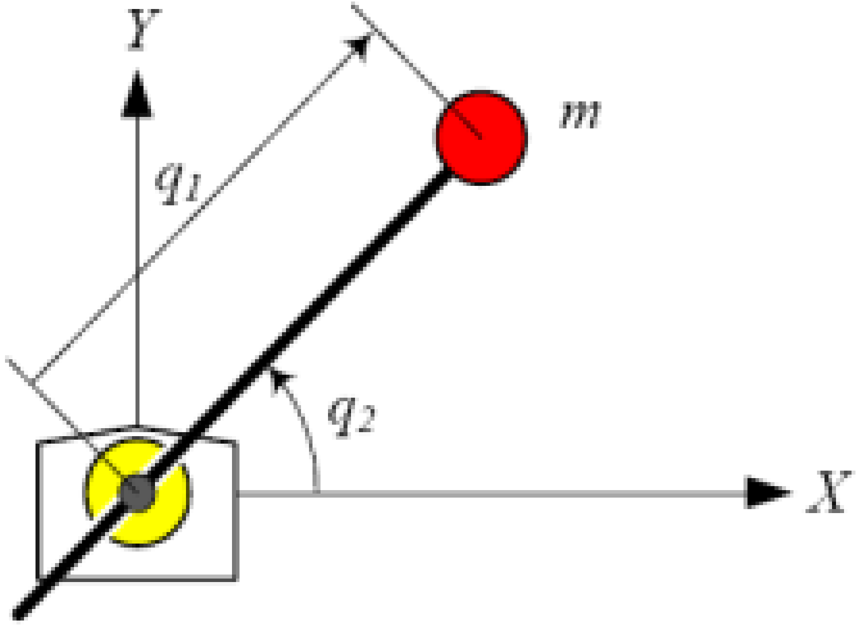
$$K = \frac{1}{2} J_\ell \dot{q}_1^2 + \frac{1}{2} J_m \dot{q}_2^2$$

$$P = Mg\ell(1 - \cos q_1) + \frac{1}{2} k(q_1 - q_2)^2$$



$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \quad \Longrightarrow \quad \begin{aligned} J_\ell \ddot{q}_1 + Mgl\sin(q_1) + k(q_1 - q_2) &= 0 \\ J_m \ddot{q}_2 + k(q_2 - q_1) &= u \end{aligned}$$

- Consider a ***planar polar manipulator*** with massless link and a massive point *m*.



- The kinetic energy of the manipulator is:

$$
\begin{aligned}
K &= \frac{1}{2} m_2 \dot{X}_2^2 + \frac{1}{2} m_2 \dot{Y}_2^2 \\
&= \frac{1}{2} m \left(\frac{d}{dt} \left(q_1 \cos q_2 \right) \right)^2 + \frac{1}{2} m \left(\frac{d}{dt} \left(q_1 \sin q_2 \right) \right)^2 \\
&= \frac{1}{2} m \left(\dot{q}_1^2 + q_1^2 \dot{q}_2^2 \right)
\end{aligned}
$$

- The potential energy of the manipulator is:

$$P = m_2 g\, Y_2 = m_2 g\, q_1\, sinq_2$$

- Therefore, the Lagrangian is:

$$\mathcal{L} = K - P = \frac{1}{2} m \left(\dot{q}_1^2 + q_1^2 \dot{q}_2^2\right) - mgq_1 \sin q_2$$

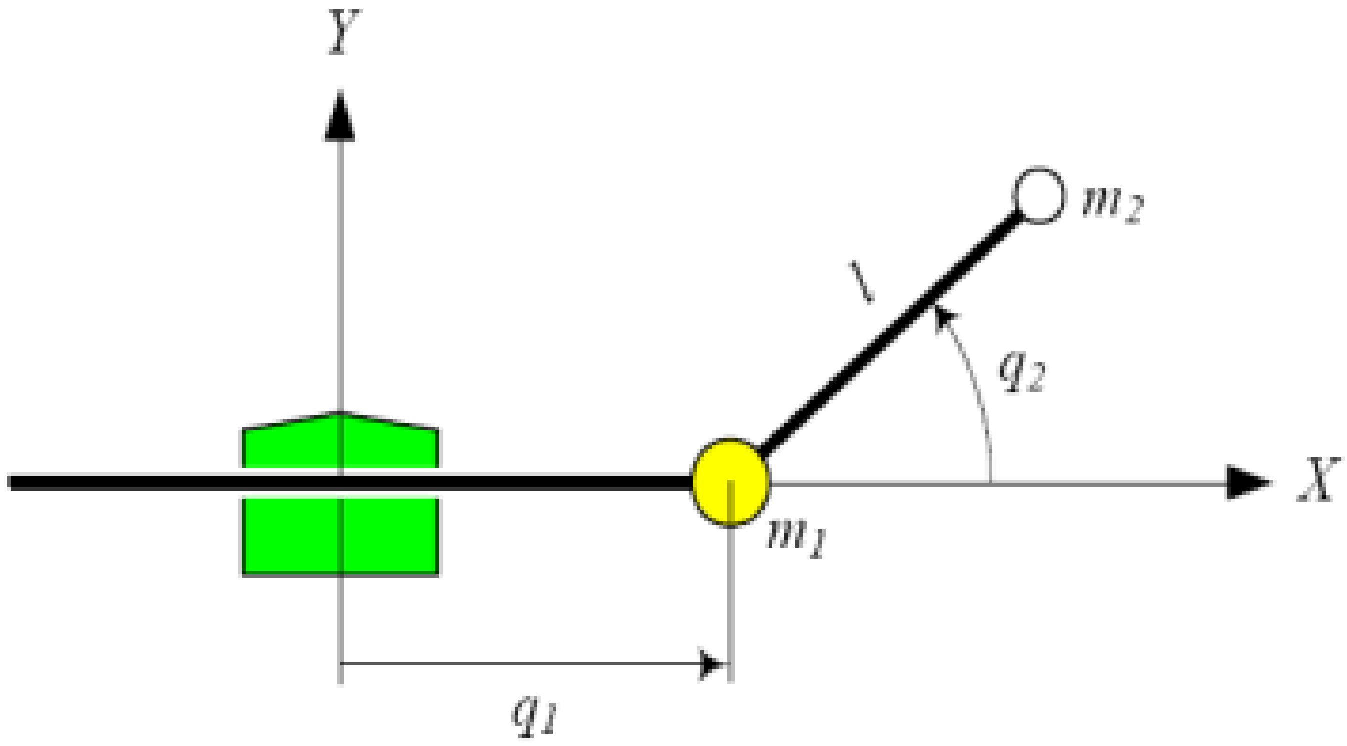
Applying the Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i \qquad i = 1, 2$$

provides the following equations of motion.

$$\begin{aligned} m\ddot{q}_1 - mq_1\dot{q}_2^2 + mg\sin q_2 &= \tau_1 \\ mq_1^2\ddot{q}_2 + 2mq_1\dot{q}_1\dot{q}_2 + mgq_1\cos q_2 &= \tau_2 \end{aligned}$$

- Consider a planar manipulator with massless links and two massive points m_1 and m_2. Determine the equations of motion.



- The kinetic energy

$$
\begin{aligned}
K_1 &= \frac{1}{2} m_1 \dot{q}_1^2 \\
K_2 &= \frac{1}{2} m_2 \dot{X}_2^2 + \frac{1}{2} m_2 \dot{Y}_2^2 \\
&= \frac{1}{2} m_2 \left(\frac{d}{dt} \left(q_1 + l \cos q_2 \right) \right)^2 + \frac{1}{2} m_2 \left(\frac{d}{dt} \left(l \sin q_2 \right) \right)^2 \\
&= \frac{1}{2} m_2 \left(\dot{q}_1 - l \dot{q}_2 \sin q_2 \right)^2 + \frac{1}{2} m_2 \left(l \dot{q}_2 \cos q_2 \right)^2 \\
&= \frac{1}{2} m_2 \left(\dot{q}_1^2 + l^2 \dot{q}_2^2 - 2 l \dot{q}_1 \dot{q}_2 \sin q_2 \right)
\end{aligned}
$$

- The potential energy of the manipulator is:

$$P = m_2 g \mathrm{Y}_2 = m_2 g\ l\ sin q_2$$

- Therefore, the Lagrangian is:

$$\mathcal{L} = K - P = K_1 + K_2 - P$$

$$= \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 \left(\dot{q}_1^2 + l^2 \dot{q}_2^2 - 2l \dot{q}_1 \dot{q}_2 \sin q_2 \right) - m_2 g l \sin q_2$$

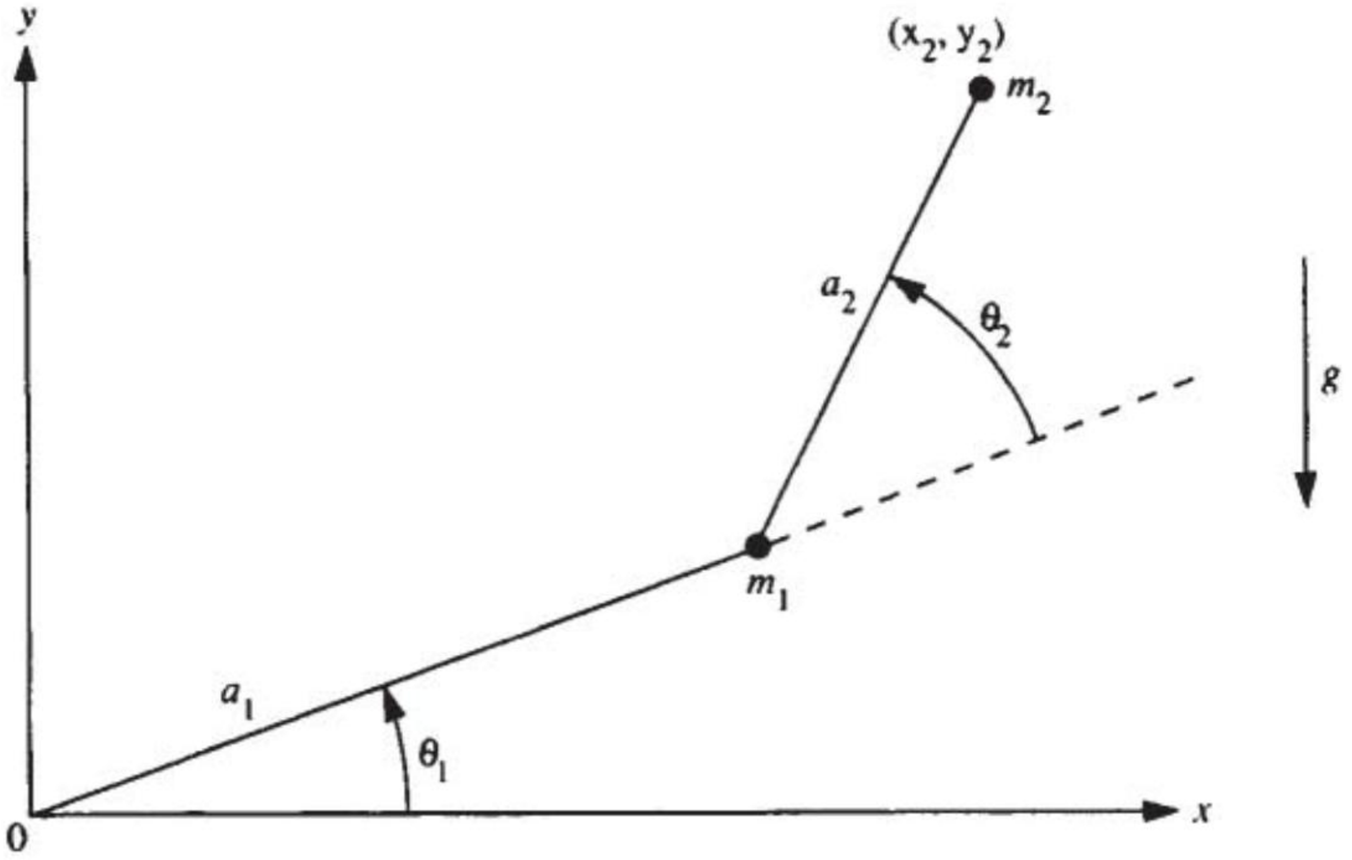
Applying the Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \boldsymbol{\tau}_i \qquad i = 1, 2$$

provides the following equations of motion.

$$\begin{aligned} (m_1 + m_2)\,\ddot{q}_1 - m_2 l \ddot{q}_2 \sin q_2 - m_2 l \dot{q}_2^2 \cos q_2 &= \boldsymbol{\tau}_1 \\ m_2 l^2 \ddot{q}_1 - m_2 l \ddot{q}_1 \sin q_2 + m_2 g l \cos q_2 &= \boldsymbol{\tau}_2 \end{aligned}$$

Example: 2DoF Robot



The joint variable is $q=[\theta_1 \ \ \theta_2]^T$

and the generalized force vector is $\tau = [\tau_1 \quad \tau_2]^T$

with τ_1, and τ_2 torques supplied by the actuators.

a. Kinetic and Potential Energy

For link 1 the kinetic and potential energies are

$$K_1 = \tfrac{1}{2} m_1 a_1^2 \dot{\theta}_1^2$$
$$P_1 = m_1 g a_1 \sin\theta_1 .$$

For link 2 we have

$$x_2 = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$$
$$y_2 = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$$
$$\dot{x}_2 = -a_1 \dot{\theta}_1 \sin\theta_1 - a_2(\dot{\theta}_1 + \dot{\theta}_2) \sin(\theta_1 + \theta_2)$$
$$\dot{y}_2 = a_1 \dot{\theta}_1 \cos\theta_1 + a_2(\dot{\theta}_1 + \dot{\theta}_2) \cos(\theta_1 + \theta_2),$$

so that the velocity squared is

$$\begin{aligned} v_2^2 &= \dot{x}_2^2 + \dot{y}_2^2 \\ &= a_1^2\dot{\theta}_1^2 + a_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2a_1a_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\cos\theta_2. \end{aligned}$$

Therefore, the kinetic energy for link 2 is

$$\begin{aligned} K_2 &= \tfrac{1}{2}m_2v_2^2 \\ &= \tfrac{1}{2}m_2a_1^2\dot{\theta}_1^2 + \tfrac{1}{2}m_2a_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 \\ &\quad + m_2a_1a_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\cos\theta_2. \end{aligned}$$

The potential energy for link 2 is

$$P_2 = m_2gy_2 = m_2g[a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2)].$$

b. Lagrange's Equation

The Lagrangian for the entire arm is

$$
\begin{aligned}
L = & K - P = K_1 + K_2 - P_1 - P_2 \\
= & \tfrac{1}{2}(m_1 + m_2)a_1^2\theta_1^2 + \tfrac{1}{2}m_2a_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 \\
& + m_2a_1a_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\cos\theta_2 \\
& - (m_1 + m_2)ga_1\sin\theta_1 \\
& - m_2ga_2\sin(\theta_1 + \theta_2).
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial \dot{\theta}_1} =&(m_1 + m_2)a_1^2\dot{\theta}_1 + m_2a_2^2(\dot{\theta}_1 + \dot{\theta}_2) \\
&+ m_2a_1a_2(2\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2 \\
\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} =&(m_1 + m_2)a_1^2\ddot{\theta}_1 \\
&+ m_2a_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2a_1a_2(2\ddot{\theta}_1 + \theta_2)\cos\theta_2 \\
&- m_2a_1a_2(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)\sin\theta_2 \\
\frac{\partial L}{\partial \theta_1} =&- (m_1 + m_2)ga_1\cos\theta_1 - m_2ga_2\cos(\theta_1 + \theta_2) \\
\frac{\partial L}{\partial \dot{\theta}_2} =&m_2a_2^2(\dot{\theta}_1 + \dot{\theta}_2) + m_2a_1a_2\dot{\theta}_1\cos\theta_2 \\
\frac{d}{dt}\frac{\partial L}{\partial \theta_2} =&m_2a_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2a_1a_2\ddot{\theta}_1\cos\theta_2 \\
&- m_2a_1a_2\dot{\theta}_1\dot{\theta}_2\sin\theta_2 \\
\frac{\partial L}{\partial \theta_2} =&- m_2a_1a_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\sin\theta_2 - m_2ga_2\cos(\theta_1 + \theta_2).
\end{aligned}
$$

Finally, according to Lagrange's equation, the arm dynamics are given by the two coupled nonlinear differential equations

$$
\begin{aligned}
\tau_1 &= \left[(m_1 + m_2)a_1^2 + m_2 a_2^2 + 2m_2 a_1 a_2 \cos\theta_2\right]\ddot{\theta}_1 \\
&\quad + \left[m_2 a_2^2 + m_2 a_1 a_2 \cos\theta_2\right]\ddot{\theta}_2 - m_2 a_1 a_2 (2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)\sin\theta_2 \\
&\quad + (m_1 + m_2) g a_1 \cos\theta_1 + m_2 g a_2 \cos(\theta_1 + \theta_2) \\
\tau_2 &= \left[m_2 a_2^2 + m_2 a_1 a_2 \cos\theta_2\right]\ddot{\theta}_1 + m_2 a_2^2 \ddot{\theta}_2 + m_2 a_1 a_2 \dot{\theta}_1^2 \sin\theta_2 \\
&\quad + m_2 g a_2 \cos(\theta_1 + \theta_2).
\end{aligned}
$$

c. Manipulator Dynamics

Writing the arm dynamics in vector form yields

$$M(q)\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}+\begin{bmatrix}-m_2a_1a_2(2\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_2^2)\sin\theta_2\\ m_2a_1a_2\dot{\theta}_1^2\sin\theta_2\end{bmatrix}$$

$$+\begin{bmatrix}m_1+m_2)ga_1\cos\theta_1+m_2ga_2\cos(\theta_1+\theta_2)\\ m_2ga_2\cos(\theta_1+\theta_2)\end{bmatrix}$$

$$=\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}.$$

where

$$M(q)=$$

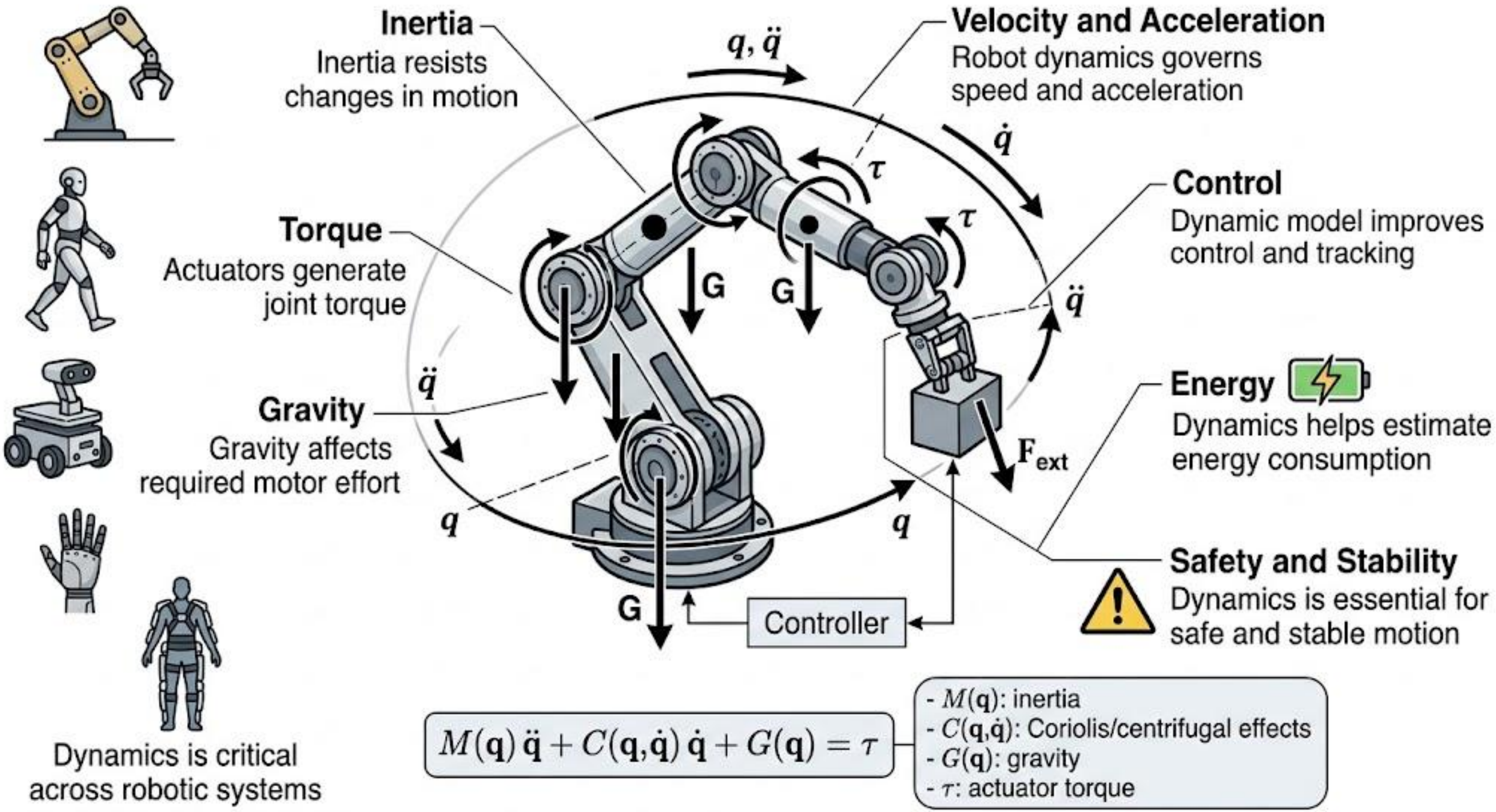
$$\begin{bmatrix}(m_1+m_2)a_1^2+m_2a_2^2+2m_2a_1a_2\cos\theta_2 & m_2a_2^2+m_2a_1a_2\cos\theta_2\\ m_2a_2^2+m_2a_1a_2\cos\theta_2 & m_2a_2^2\end{bmatrix}$$

These manipulator dynamics are in the standard form

$$M(q)\ddot{q} + V(q, \dot{q}) + G(q) = \tau$$

with $M(q)$ the inertia matrix, $V(q,\dot{\mathbf{q}})$ the Coriolis/centripetal vector, and $G(q)$ the gravity vector. Note that $M(q)$ is symmetric.

$$\boxed{D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) \quad = \quad \tau}$$



Inertia
Inertia resists changes in motion
$q, \ddot{q}$
Velocity and Acceleration
Robot dynamics governs speed and acceleration
$\dot{q}$
τ
τ
Torque
Actuators generate joint torque
G
G
Control
Dynamic model improves control and tracking
$\ddot{q}$
$\ddot{q}$
Gravity
Gravity affects required motor effort
Energy
Dynamics helps estimate energy consumption
F_{ext}
q
q
Safety and Stability
Dynamics is essential for safe and stable motion
G
Controller
Dynamics is critical across robotic systems
$M(\mathbf{q})\,\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} + G(\mathbf{q}) = \tau$
- $M(\mathbf{q})$: inertia
- $C(\mathbf{q},\dot{\mathbf{q}})$: Coriolis/centrifugal effects
- $G(\mathbf{q})$: gravity
- τ: actuator torque

Questions

Thank You