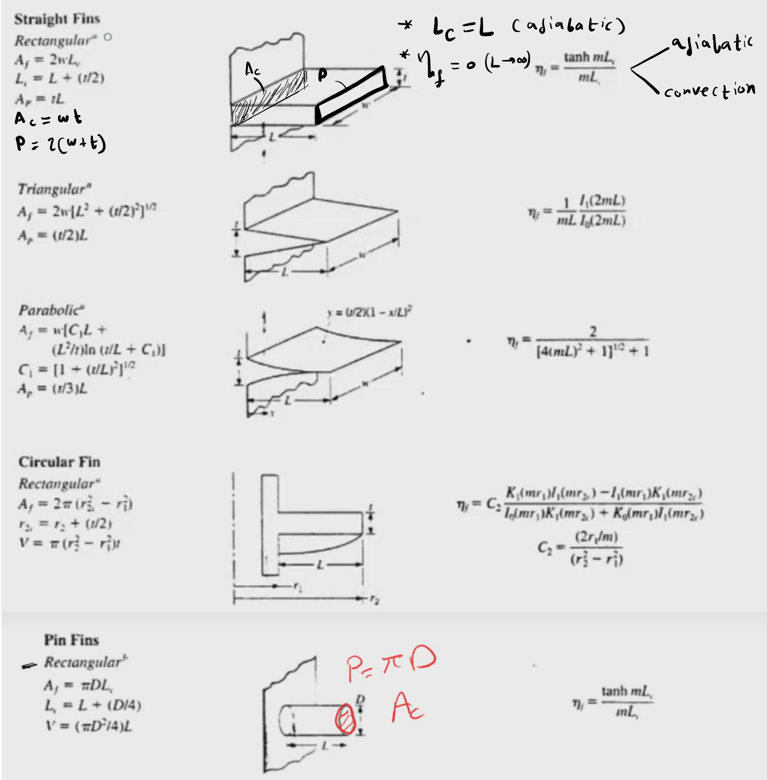
1. Geometry

a) Parameters   b) Efficiency



Straight Fins

Rectangular[a]

$A_f = 2wL_c$
$L_c = L + (t/2)$
$A_p = tL$
$A_c = wt$
$P = 2(w+t)$

* $L_c = L$ (adiabatic)
* $\eta_f = 0\ (L \to \infty)$

$\eta_f = \dfrac{\tanh mL_c}{mL_c}$ — adiabatic / convection

Triangular[a]

$A_f = 2w[L^2 + (t/2)^2]^{1/2}$
$A_p = (t/2)L$

$\eta_f = \dfrac{1}{mL}\dfrac{I_1(2mL)}{I_0(2mL)}$

Parabolic[a]

$A_f = w[C_1 L + (L^2/t)\ln(t/L + C_1)]$
$C_1 = [1 + (t/L)^2]^{1/2}$
$A_p = (t/3)L$

$y = (t/2)(1 - x/L)^2$

$\eta_f = \dfrac{2}{[4(mL)^2 + 1]^{1/2} + 1}$

Circular Fin

Rectangular[a]

$A_f = 2\pi(r_{2c}^2 - r_1^2)$
$r_{2c} = r_2 + (t/2)$
$V = \pi(r_2^2 - r_1^2)t$

$\eta_f = C_2 \dfrac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$

$C_2 = \dfrac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$

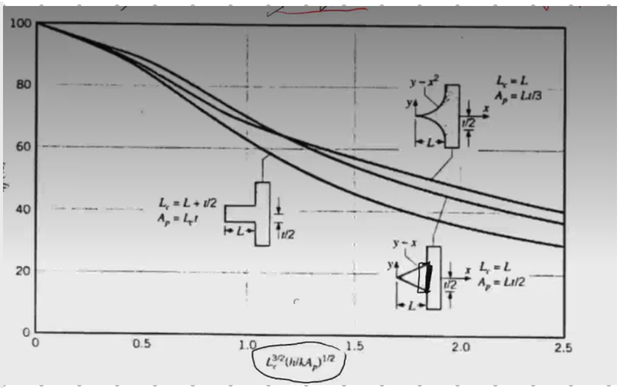
Pin Fins

Rectangular[b]

$A_f = \pi D L_c$
$L_c = L + (D/4)$
$V = (\pi D^2/4)L$

$P = \pi D$, A_c

$\eta_f = \dfrac{\tanh mL_c}{mL_c}$



2. Thermal Properties

$k, h, T \begin{cases} T_b \\ T_\infty \\ T_f \end{cases}$

3. Tip Condition

a) Temperature Distributaion b) Fin Heat Transfer Rate

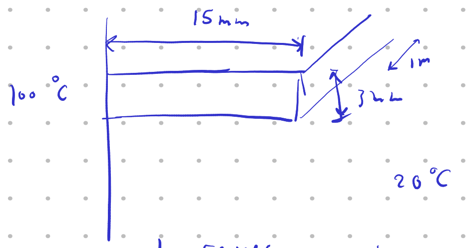
$\eta = \dfrac{q_f}{A_f \times h \times \Delta T} < 1$ ($\Delta T = T_b - T_\infty$; $t \times w$)

$\dfrac{\theta}{\theta_b} = \dfrac{T - T_\infty}{T_b - T_\infty}$

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\|_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic $d\theta/dx\|_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $m^2 \equiv hP/kA_c$
$\theta_b = \theta(0) = T_b - T_\infty$ $M \equiv \sqrt{hPkA_c}\,\theta_b = \sqrt{hPkA_c}\ \theta_b$

2. A straight fin fabricated from 2024 aluminum alloy (k= 185 W/m.K.) has a base thickness of t=3mm and a length of L = 15mm. Its base temperature is $T_b = 100°C$ and it is exposed to a fluid for which $T_\infty = 20°C$ and $h = 50\ W/m^2.K$. For the previous conditions and for a fin of unit width, determine the heat transfer rate, fin effectiveness and fin efficiency using the following boundary conditions, (a) infinitely long fin, (b) adiabatic fin tip, (c) fin with tip temperature of 50°C and (d) convection from the fin tip.



$h = 50\ W/m^2 K$ $k = 185\ W/mk$

$Q,\ \varepsilon,\ \eta$

a) $q_f = M = \sqrt{hPkA_c}\ \theta_b = 596.9\ W$

$P = 2(w+t) = 2.006\ m$

$A_c = wt = 3\times10^{-3}\ m^2$

$\theta_b = T_b - T_\infty = 80\ °C$

$\varepsilon = \dfrac{q_f}{q} = \dfrac{596.9}{12} = 49.74$

$q = hA_{c,b}(T_b - T_\infty) = 12\ W$

$A_{c,b} = wt = 3\times10^{-3}\ m^2$

$\eta = \dfrac{q_f}{q_f^*} \rightarrow 0$

$q_f^* = hA_f(T_b - T_\infty) \rightarrow \infty$ as $A_f \rightarrow \infty$

b) $q_f = M \tanh mL = 118.73\ W$

$m = \sqrt{\dfrac{hP}{kA_c}} = 13.44$

$\varepsilon = \dfrac{q_f}{q} = 9.89$

$\eta_f = \dfrac{\tanh mL_c}{mL_c} = 98.7\ \%$

$L_c = L$

c) $q_f = M \dfrac{\cosh mL - \frac{\theta_L}{\theta_b}}{\sinh mL} = 189.8\ W$

$\theta_L = T - T_\infty = 30\ °C$

$\varepsilon = \dfrac{q_f}{q} = 158.167\ W$

$\eta_f = \dfrac{q_f}{q_f^*} > 1$ (Not applicable)

$q_f^* = hA_f(T_b - T_\infty) = 132\ W$

$A_f = 2wL_c = 0.033\ m^2$

$L_c = L + t/2 = 16.5\times10^{-3}\ m$

d) $q_f = M \dfrac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL} = 130.21\ W$

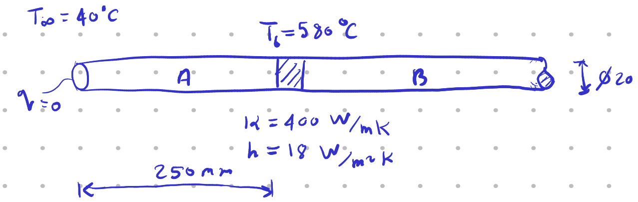
$\varepsilon_f = \dfrac{q_f}{q} = 10.85$

$\eta_f = \dfrac{\tanh mL_c}{mL_c} = 98.4\ \%$

$L_c = L + t/2 = 16.5\times10^{-3}\ m$

4. Two copper rods A and B of the same material (k= 400W/m.K) having same diameter of 20 mm but different lengths. The two rods are to be soldered together end to end with soldering material having melting point of 580°C. The two rods are exposed to ambient air at temperature of 40°C with convective coefficient 18 W/m^2.K, if rod A has finite length of 250 mm with adiabatic tip, while rod B is very long, calculate the minimum power needed to achieve the soldering and the tip temperature of rod A.

$T_b = 580°C$



Rod A:

$$q_f = M \tanh mL = 129.475 \text{ W}$$

$$M = \sqrt{hPKA_c} \quad \theta_b = 203.85 \text{ W}$$

$$P = \pi d = 0.063 \text{ m}$$

$$A_c = \frac{\pi}{4} d^2 = 3.1416 \times 10^{-4} \text{ m}^2$$

$$\theta_b = T_b - T_\infty = 540 \text{ °C}$$

$$m = \sqrt{\frac{hP}{kA_c}} = 3$$

$$\frac{\theta_{tip}}{\theta_b} = \frac{T_{tip} - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL} = 0.772 \longrightarrow T_{tip} = 457.1 \text{ °C} \quad ※2$$

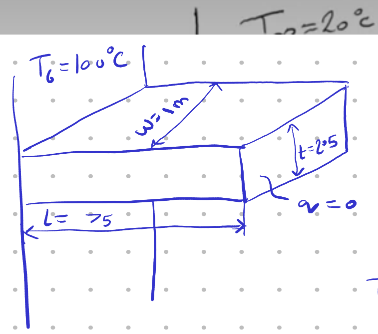
$$x_{tip} = L$$

Rod B:

$$q_f = M = 203.85 \text{ W}$$

$$q_{min} = q_{f_A} + q_{f_B} = 333.325 \text{ W} \quad ※1$$

7. A straight fin of rectangular profile is made of stainless steel (k= 16.3 W/m.K) has a length of 75 mm and 2.5 mm thickness. The base temperature is maintained at 100°C while the fin is exposed to ambient temperature T_∞ = 20°C with h= 45 W/m^2.K. Calculate the (1) heat loss by the fin per meter width, the (2) fin effectiveness. Also calculate the (3) tip temperature of the fins considering the end is adiabatic tip.



$k = 16.3$ W/mK
$h = 45$ W/m^{2}K

$T_\infty = 20°C$

$q_f = M \tanh mL = 153.14$ W ※1

$M = \sqrt{hPkA_c}\ \theta_b = 153.4$

$P = 2(t + w) = 2.005$ m

$A_c = wt = 2.5 \times 10^{-3}$ m^2

$\theta_b = T_b - T_\infty = 80°c$

$m = \sqrt{\dfrac{hP}{kA_c}} = 47.05$

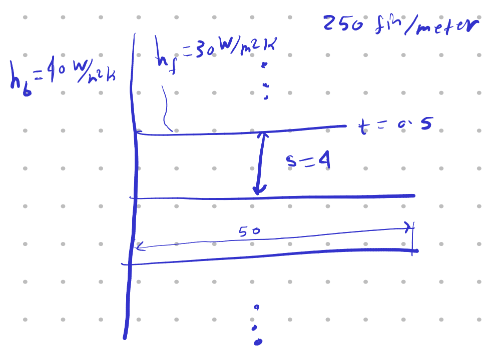
$\varepsilon = \dfrac{q_f}{q} = 17.016$

$q = h A_{c,b} (T_b - T_\infty) = 9$ W ※2

$\dfrac{\theta_{tip}}{\theta_b} = \dfrac{\cosh m(L - x_{tip})}{\cosh mL} = \dfrac{T_{tip} - T_\infty}{T_b - T_\infty} = 0.059 \longrightarrow T_{tip} = 24.69°C$ ※3

$x_{tip} = L$

8. Determine the percentage increase in heat transfer rate associated with attaching aluminum fins (k=237 W/m.K) of rectangular profile to plane wall. The fins are 50mm long, 0.5mm thick and are equally spaced at distance of 4mm (250 fins/meter). The convective coefficient with the base wall is 40 W/m^2.K and with fins is 30 W/m^2.K.



$$q = h_b A (T_b - T_\infty) = 40\,\theta_b$$

$$A = 1 \times 1 = 1m^2$$

$$q_f = M \frac{\sinh mL + \frac{h_f}{mk}\cosh mL}{\cosh mL + \frac{h_f}{mk}\sinh mL} = C_2\theta_b$$

$$M = \sqrt{h_f P k A_c}\;\theta_b = C_1\theta_b$$

$$m = \sqrt{\frac{h_f P}{k A_c}} = \checkmark$$

$$P = 2(w+t) = \checkmark$$

$$A_c = wt = \checkmark$$

$$\%_{inc} = \frac{q_t - q}{q} \times 100 = \frac{C_4 - 40}{40} \times 100 = \checkmark$$

$$q_t = N_f q_f + q_b = N_f C_2\theta_b + C_3\theta_b = C_4\theta_b$$

$$N_f = 250 \quad \text{(we are taking 1m height)}$$

$$q_b = h_b A_b \theta_b = C_3\theta_b$$

$$A_b = 1 \times [1 - 250t] = \checkmark$$